“Fórmulas, Propriedades e Somatórios úteis” - Prof. Valdísio Viana

Edição: Italo Maia

01. $\sum_{k=1}^{n}(c.a_k+b_k)=c.\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k \quad \rightarrow \quad \sum_{k=1}^{n}\Theta[f(k)]=\Theta[\sum_{k=1}^{n}f(k)]$

02. $\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$ 03. $\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$

04. $\sum_{k=1}^{n}k^3=[\sum_{k=1}^{n}k]^2=\frac{n^2(n+1)^2}{4}$ 05. $\sum_{k=1}^{n}k^4=\frac{n(n+1)(9n^2+n+1)}{30}$ 06. $\sum_{k=1}^{n}k^p\approx\Theta(N^{p+1})$

07. $\sum_{k=1}^{n}x^k=\frac{x^{n+1}-1}{x-1}$ (PG finita de razão $x\neq 1$)

08. $\sum_{k=0}^{\infty}x^k=\frac{1}{1-x}$ (PG infinita de razão $|x|<1$)

09. $\sum_{k=1}^{n}\frac{1}{k}=\ln(n)+O(1)$ 10. $\sum_{k=1}^{\infty}k.x^k=\frac{x}{(1-x)^2}$ para $|x|<1$

11. $\sum_{k=1}^{n}k.x^k=\frac{x-(n+1)x^{n+1}+n.x^{n+2}}{(1-x)^2}$ para $x\neq 1$

12. $\sum_{k=1}^{n}k.2^k=(n-1).2^{n+1}+2$

13. $\sum_{k=1}^{n-1}\frac{1}{k(k+1)}=\sum_{k=1}^{n-1}\frac{1}{k}-\sum_{k=1}^{n-1}\frac{1}{k+1}$

14. a) $\sum_{k=1}^{n}\binom{n}{k}=2^n$ b) $\sum_{k=1}^{n}\binom{k}{m}=\binom{n+1}{m+1}$ c) $\max_{1\leq k\leq n}\left\{\binom{n}{k}\right\}=\binom{n}{\lceil k/2\rceil}$

15.

a)Partições $\{A_1, A_2 \ldots A_k\}$ de A → $\left\{{n \atop k}\right\}=k\left\{{n-1 \atop k}\right\}+\left\{{n-1 \atop k-1}\right\}$

b) $N!=\sqrt{2\pi N}\left(\frac{N}{e}\right)^2$

16. $S(n)=c.S(n-1)+g(n)\rightarrow S(n)=c^{n-1}S(1)+\sum_{i=2}^{n}c^{n-i}g(i)$

17. $T(n)=a.T(n/b)+f(n)\rightarrow T(n)=\Theta\left(n^{\log_b^a}\right)+\sum_{i=0}^{h-1}a^i f(\frac{n}{b^i})$ para $h=\log_b^n$

18.

Equação homogênea característica

$$a_0 T(n) + a_1 T(n-1) + a_2 T(n-2) + ... + a_k T(n-k) = 0$$

Raízes distintas

$$r_1 \neq r_2 \neq r_3 ... \neq r_p \rightarrow T(n) = c_1 r_1^N + c_2 r_2^N ... c_p r_p^N$$

Raízes múltiplas

$$r_1 = r_2 = r_3 ... = r_p = r \rightarrow T(n) = c_1 r^N + c_2 N r^N + c_3 N^2 r^N ... c_p N^{p-1} r^N$$

Raízes Complexas Distintas

$$r = a \pm bi \rightarrow T(n) = c.p^N \quad \text{onde} \quad p = \sqrt{a^2 + b^2}$$

Raízes complexas múltiplas

$$r_1 = r_2 = a \pm bi \rightarrow T(n) = c_1 p^N + c_2 N p^N \quad \text{onde} \quad p = \sqrt{a^2 + b^2}$$

19. $lg(N) = \log_2^N$

20. $f(n) = \log_a^N \rightarrow O(f) = O[lg(N)] \forall a \in \Re^+, a \neq 1$